No part of this product may be reproduced in any form or by any electronic or mechanical means, including information storage and retrieval systems, without written permission from the IB.

Additionally, the license tied with this product prohibits commercial use of any selected files or extracts from this product. Use by third parties, including but not limited to publishers, private teachers, tutoring or study services, preparatory schools, vendors operating curriculum mapping services or teacher resource digital platforms and app developers, is not permitted and is subject to the IB's prior written consent via a license. More information on how to request a license can be obtained from http://www.ibo.org/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license.

Aucune partie de ce produit ne peut être reproduite sous quelque forme ni par quelque moyen que ce soit, électronique ou mécanique, y compris des systèmes de stockage et de récupération d'informations, sans l'autorisation écrite de l'IB.

De plus, la licence associée à ce produit interdit toute utilisation commerciale de tout fichier ou extrait sélectionné dans ce produit. L'utilisation par des tiers, y compris, sans toutefois s'y limiter, des éditeurs, des professeurs particuliers, des services de tutorat ou d'aide aux études, des établissements de préparation à l'enseignement supérieur, des fournisseurs de services de planification des programmes d'études, des gestionnaires de plateformes pédagogiques en ligne, et des développeurs d'applications, n'est pas autorisée et est soumise au consentement écrit préalable de l'IB par l'intermédiaire d'une licence. Pour plus d'informations sur la procédure à suivre pour demander une licence, rendez-vous à l'adresse http://www.ibo.org/fr/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license.

No se podrá reproducir ninguna parte de este producto de ninguna forma ni por ningún medio electrónico o mecánico, incluidos los sistemas de almacenamiento y recuperación de información, sin que medie la autorización escrita del IB.

Además, la licencia vinculada a este producto prohíbe el uso con fines comerciales de todo archivo o fragmento seleccionado de este producto. El uso por parte de terceros —lo que incluye, a título enunciativo, editoriales, profesores particulares, servicios de apoyo académico o ayuda para el estudio, colegios preparatorios, desarrolladores de aplicaciones y entidades que presten servicios de planificación curricular u ofrezcan recursos para docentes mediante plataformas digitales— no está permitido y estará sujeto al otorgamiento previo de una licencia escrita por parte del IB. En este enlace encontrará más información sobre cómo solicitar una licencia: http://www.ibo.org/es/contact-the-ib/media-inquiries/for-publishers/guidance-for-third-party-publishers-and-providers/how-to-apply-for-a-license.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 3 – sets, relations and groups

Wednesday 15 May 2019 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

2219 – 7209
© International Baccalaureate Organization 2019

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

The relation R is defined on $\mathbb{N}$ by $xRy \Leftrightarrow x^2 + y^2 \equiv 0 \pmod 2$.

(a) Show that R is an equivalence relation. [7]

(b) Determine the equivalence classes. [2]

2. [Maximum mark: 14]

Binary operations $\circ$ and $*$ are defined on the set of complex numbers such that $z_1 \circ z_2 = A(z_1 + z_2)$ and $z_1 * z_2 = Bz_1z_2$ where A and B are real non-zero constants.

(a) Determine the identity with respect to $*$. [2]

(b) Show that every element of the set of complex numbers, apart from $z = 0$, has an inverse with respect to $*$. [3]

(c) Find the value of A for which the operation $\circ$ is associative. [4]

(d) Show that $*$ is distributive over $\circ$ for all values of A and B. [5]

3. [Maximum mark: 9]

A non-Abelian group, $\{G, \circ\}$, has eight elements $e, a, a^2, a^3, b, ab, a^2b, a^3b$, where e is the identity. The binary operation is $\circ$ and, for example, $a \circ b$ is denoted by ab.
The element a has order 4, the element b has order 2 and $ba = a^3b$.

(a) Prove that

(i) $ba^2 = a^2b$;

(ii) $ba^3 = ab$. [4]

(b) The group $\{G, \circ\}$ has the following Cayley table. Find the elements $\boldsymbol{P, Q, R, S, T, U, V, W}$ of the group.

$\circ$	e	a	a^2	a^3	b	ab	a^2b	a^3b
e	e	a	a^2	a^3	b	ab	a^2b	a^3b
a	a	a^2	a^3	e	ab	a^2b	a^3b	b
a^2	a^2	a^3	e	a	a^2b	a^3b	b	ab
a^3	a^3	e	a	a^2	a^3b	b	ab	a^2b
b	b	a^3b	a^2b	ab	e	a^3	a^2	a
ab	ab	b	a^3b	a^2b	a	e	$\boldsymbol{R}$	$\boldsymbol{U}$
a^2b	a^2b	ab	b	$\boldsymbol{P}$	a^2	a	$\boldsymbol{S}$	$\boldsymbol{V}$
a^3b	a^3b	a^2b	ab	$\boldsymbol{Q}$	a^3	a^2	$\boldsymbol{T}$	$\boldsymbol{W}$

[4]

(c) State why $\{G, \circ\}$ is not isomorphic to the group $\{\{0, 1, 2, 3, 4, 5, 6, 7\}, +_8\}$, where $+_8$ denotes addition modulo 8. [1]

4. [Maximum mark: 7]

Let S be the set of real numbers of the form $a + b\sqrt{5}$ where $a, b \in \mathbb{Q}$, a^2 cannot equal $5b^2$, and a, b are not simultaneously zero. Show that S is a group under the operation of multiplication. Associativity may be assumed. [7]

5. [Maximum mark: 11]

Consider the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (x + y, x - y)$.

(a) Prove that the function f is a bijection. [9]

Consider the function $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by $g(x, y) = (x + y, x - y)$.

(b) Explain why the function g is not a bijection. [2]